For the center of gravity of the *curved* surface we have,

$$\bar{x}=\frac{4\int 2xyds}{S}, =\frac{8r\int_0^r xdx}{8r^2}=\frac{1}{r}\int_0^r xdx, =\tfrac{1}{2}r, =\tfrac{1}{4}a.$$

For the center of gravity of the *whole* surface, since the curved surface is *twice* that of the base we have, $\bar{x}=\frac{2}{3}.\frac{1}{4}a=\frac{1}{6}a$.

Also solved by *H. C. WHITAKER*, *C. W. M. BLACK*, and the *PROPOSER*.

Professors Black and Scheffer used "side$=2a$" as in Problem 47, instead of *side*$=a$, and hence their results did not agree with those in the published solutions. The results obtained were: Volume$=8a^3/3$, surface$=8a^2$, center of gravity of volume$=3a/8$, and center of gravity of surface$=\frac{1}{2}a$. See problem 42 for two additional SOLUTIONS for surface and volume. EDITOR.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

38. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

A prolate spheroid of revolution is fixed at its focus; a blow is given it at the extremity of the axis minor in a line tangent to the direction perpendicular to the axis major. Find the axis about which the body begins to rotate. [From *Loudon's Rigid Dynamics*.]

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

The general equations of motion are:

$$\left.\begin{array}{l} A\omega_x-(\Sigma mxy)\omega_y-(\Sigma mxz)\omega_z=L \\ B\omega_y-(\Sigma myz)\omega_z-(\Sigma myx)\omega_x=M \\ C\omega_z-(\Sigma mzx)\omega_x-(\Sigma mzy)\omega_y=N \end{array}\right\}\dots\dots\dots\dots\dots (1).$$

The equation to the ellipsoid with focus as origin is $a^2y^2+a^2z^2+b^2x^2=2aeb^2x+b^4$. Now $\Sigma mxy=\Sigma mxz=\Sigma myz=0$. $\therefore$ (1) reduce to

$$\left.\begin{array}{l} A\omega_x=L \\ B\omega_y=M \\ C\omega_z=N \end{array}\right\}\dots\dots\dots\dots\dots\dots\dots (2).$$

Let $2aeb^2x+b^4-b^2x^2=a^2c^2$. Then

$$A=\Sigma m(y^2+z^2)=\mu\int_{-a(1-e)}^{a(1+e)}\int_{-c}^{c}\int_{-\sqrt{c^2-y^2}}^{\sqrt{c^2-y^2}}(y^2+z^2)dxdydz$$

$$=\frac{4\mu}{3}\int_{-a(1-e)}^{a(1+e)}\int_{0}^{c}\{3y^2\sqrt{c^2-y^2}+(c^2-y^2)\sqrt{c^2-y^2}\}dxdy$$

$$=\frac{\pi\mu}{2a^4}\int_{-a(1-e)}^{a(1+e)}(2aeb^2x+b^4-b^2x^2)^2dx=\tfrac{8}{15}\mu\pi ab^4.$$

$$B=C=\Sigma m(x^2+y^2)=\mu\int_{-a(1-e)}^{a(1+e)}\int_{-c}^{c}\int_{-\sqrt{c^2-y^2}}^{\sqrt{c^2-y^2}}(x^2+y^2)dxdydz$$

$$=4\mu\int_{-a(1-e)}^{a(1+e)}\int_{0}^{c}(x^2+y^2)\sqrt{c^2-y^2}\,dxdy$$

$$=\frac{\mu\pi}{4}\int_{-a(1-e)}^{a(1+e)}(4c^2x^2+c^4)dx=\tfrac{8}{15}\mu\pi a^3b^2(1+2e^2).$$

Let the blow $=P$ be struck perpendicular to the plane (xy), then the moments of the impulsive forces about the axes are $L=Pb$, $M=Pae$, $N=0$.

These in (2) give

$$\left.\begin{array}{l}\tfrac{8}{15}\mu\pi ab^4\omega_x=Pb\\ \tfrac{8}{15}\mu\pi a^3b^2(1+2e^2)\omega_y=Pae\\ \omega_z=0\end{array}\right\}\cdots\cdots\cdots\cdots (3).$$

$$\therefore \frac{\omega_y}{\omega_x}=\frac{e^2}{1+2e^2}\cdot\frac{b}{ae}.$$

Let F be the focus, O the center of the ellipsoid. Then on the minor axis in the plane (xy), take $OE=\dfrac{e^2}{1+2e^2}.b$, then will FE be the axis required.

Let $a=5$, $b=4$. $\therefore e=\frac{3}{5}$. $\therefore OE=\frac{9}{43}b=\frac{36}{43}$. The resultant angular velocity will be

$$\frac{15P}{8\mu\pi a^2b^3e}.OF=\frac{P}{512\mu\pi}.OF=\frac{3P\sqrt{1993}}{22016\mu\pi},\text{ when } a=5,\ b=4.$$